Adventures in Predicting Gravitational Waves from Binary Black Holes

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This paper deals with the period that reaches from Einstein's discovery of the theory of general relativity to the development of accurate analytical descriptions of the gravitational-wave (GW) signal emitted by binary systems composed of compact objects. In November 1915, Albert Einstein [1] finalized the construction of the theory of general relativity. General relativity is both a new theory of the structure of space-time (generalizing Einstein's 1905 theory of special relativity), and a new theory of gravitation (generalizing Newton's 1687 universal law of gravitation). Einstein's theory of general relativity revolutionized the basic physical concepts of space, time, and matter, and led to the introduction of new physical objects, notably gravitational waves, black holes, and cosmological space-times.

The main characters of the scientific adventure that we wish to present today are gravitational waves (GW) and black holes (BH). Although these new concepts made their first appearance immediately after the discovery of general relativity [2–4], it took about fifty years of conceptual and observational work to fully apprehend them, and to understand that they described new physical objects which existed and could be detected in our universe.

To simplify and shorten a long history, Einstein studied the radiation of energy in the form of GWs by general moving mass distributions in 1918, with a generalization to possibly self-gravitating systems (such as binary systems) by Landau and Lifshitz in the second volume of their famous treatise of theoretical physics (first published in 1941). However, the first editions of Landau and Lifshitz's book – up to the third English edition of 1971) [5], and the French edition that Thibaut Damour had read as a young scientist – contained the following pessimistic comment on the derivation of the energy lost to GWs:

> It is necessary to note that the numerical value of this energy loss, even for astronomical objects, is so small that its effects on the motion, even over cosmic time intervals, is completely negligible (thus, for double stars, the energy loss in a year turns out to be ∼ 10−12 of the total energy).

In other words, up to the early 1970s, the theoretical physics community tended to think that the radiation of GWs by known (terrestrial or) astronomical objects was not great enough to lead to observable effects. This pessimistic view changed in the 1970s through the works of several experimental physicists and one theoretical physicist. First, Joseph Weber [6] showed that passing GWs could be detected through the excitation of resonant acoustic vibrations in aluminum cylinders, and so he constructed such detectors. Second, Freeman Dyson (of Quantum-Field-Theory fame) was the first physicist to realize (as early as 1963) that binary systems made of sufficiently compact objects (neutron stars or black holes) would (in reaction to the emission of

GWs) slowly get closer and closer, and, therefore, would slowly emit more and more GWs. He wrote [7]:

> The loss of energy by gravitational radiation will bring the two stars closer with ever-increasing speed, until, in the last second of their lives they plunge together and release a gravitational flash at a frequency of about 200 cycles and of unimaginable intensity.

And he added: «It would seem worthwhile to maintain a watch for events of this kind, using Weber's equipment or some suitable modification of it.»

These pioneering (and prophetic) works of Weber and Dyson did not attract much attention until 1969, when Weber announced that he had detected GW signals [8]. This announcement triggered a lot of activity, both on the experimental and theoretical sides. On the experimental side, an important work was Rainer Weiss's study [9] of a different type of GW detector, namely «to place free masses at several locations and measure their separations interferometrically». Weiss's detailed noise analysis showed that such interferometric detectors had very promising properties. In the following decades, several experimental groups (after failing to confirm the claimed GW detection by Weber) developed either more sensitive (cryogenic) versions of Weber's resonant detectors, or (non-resonant) interferometric detectors. On the theoretical side, many aspects of the GW signals emitted by various sources were intensely studied. In particular, it was understood (by studying the GWs emitted by test particles falling into a black hole [10, 11]), that mergers involving black holes were likely to generate GW signals whose endings will contain the characteristic quasinormal modes [12] of perturbed black holes [13]. As a young postdoc, Thibaut Damour was lucky to be in Princeton during the years 1974-1976, and to learn some of the basic aspects of the GW signals generated by test particles moving around (or falling into) black holes directly from Remo Ruffini.

Separately from the activity triggered by Weber's pioneering work, the announcement of another experimental discovery played a crucial role in GW physics. In October 1974, Russell Hulse and Joseph Taylor announced the discovery of the first binary pulsar, PSR 1913+16 [14]. The discovery of this system has been important in many ways. First, it provided the first example of the existence of binary systems made of two neutron stars of the type envisaged by Freeman Dyson. This provided strong motivation for developing detectors that could pick up the GW signals emitted by such compact binary systems[1] (especially the near-end part of these signals, comprising the rather intense GW signal emitted during the last minutes, and the final «gravitational flash» emitted during the merger). Second, the precise monitoring of the times of arrival of the radio pulses emitted by the binary pulsar PSR 1913+16 led to the first experimental proof of the reality of gravitational radiation. Indeed, Taylor and his collaborators announced as early as 1979 ([15] and see also Ref. [16]) that they could measure the secular decrease of the orbital period of this binary system, and that this

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[1] The expression «compact binary system» indicates a system composed of two compact objects. The latter refer to either of the two possible, most compact end points of stellar evolutions: neutron stars or black holes.

measurement was in agreement with the corresponding radiation-damping effect expected in general relativity.

Very soon after his arrival as a postdoc in Princeton, Damour was lucky to hear about the discovery of the first binary pulsar immediately, during one of the «Bahcall lunches» at the Institute for Advanced Study. This had an important impact on his career, leading him to work on the general relativistic theory of binary systems,[2] including both conservative and radiation-damping aspects (see Ref. [17] for a review of the theoretical work that confirmed that GW damping in general relativity predicts the observed orbital period change, as well as a review of the other confirmations of general relativity derived from binary pulsar observations).

Around 1980, the theoretical physics and astrophysics, communities became optimistic about the possibility of detecting GW signals, and began discussing estimates of the various astrophysically-generated GWs bathing the Earth [18]. Damour's own involvement in GW research was boosted by several encounters he had in 1979-1982. In particular, in July 1979, during the second Marcel Grossmann Meeting (in Trieste), he met Alain Brillet (who became a friend), and witnessed Brillet's first interactions with several European scientists involved in developing interferometric GW detectors (notably, Ronald Drever, Albrecht Rudiger, and Roland Schilling). Other important encounters

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[2] In fact, Damour had already been fascinated in his teens by the 1938 work of Einstein, Infeld and Hoffmann (EIH) on binary systems. Soon after his arrival in Princeton, he spent an afternoon with Helen Dukas at Einstein's house on Mercer Street, and she lent him the original manuscript giving the details of the EIH computations.

happened in June 1982 during the Les Houches School of Theoretical Physics conference on Gravitational Radiation (organized by Nathalie Deruelle and Tsvi Piran). There, Damour, who was looking for new scientific challenges, presented the results of the first derivation of the complete, two-and-a-half post-Newtonian (2.5-PN, i.e., 1/c5-accurate) equations of motion of a binary system in General Relativity [19]. By listening to Kip Thorne's introductory review to the theory of gravitational radiation (based on Ref. [20]), Damour understood that there was room for developing new methods for analytically computing the GW emission of generic material systems, and, notably, of binary systems. This led to the multi-year development of the Multipolar Post-Minkowskian formalism [21], in collaboration with Luc Blanchet, Bala Iyer, and others. This formalism has been instrumental for computing the GW emission of binary systems [22–24] (see also [25]) with high perturbative accuracy (much beyond the leading-order results of Einstein-Landau-Lifshitz). Damour also vividly remembers a fascinating lecture by Ron Drever during the June 1982 Les Houches School conference in which he explained the idea of recycling light to increase interferometer sensitivity [26].

During the 1980s and early 1990s, two experimental projects for constructing kilometer-scale interferometric detectors of GWs were proposed,[3] studied, and finally approved: (i) in the United States, the Laser Interferometer Gravitational-Wave Observatory (LIGO) project

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[3] For example, Alain Brillet submitted his first project to study a kilometer-size gravitational antenna to the French Institut National d'Astronomie et de G´eophysique in October 1982. This project was favorably approved in December 1982 by an ad hoc committee (of which Thibaut Damour was the rapporteur).

(founded by Ron Drever, Kip Thorne and Rai Weiss), comprising two separate 4km interferometers; and (ii) in Europe, the French-Italian Virgo project (founded by Alain Brillet and Adalberto Giazzotto). The start of these experimental projects motivated a renewed theoretical effort to improve the accuracy of the knowledge and description of the GW signals emitted by potential astrophysical GW sources.

It was slowly realized that the only type of GW sources whose existence in nature was assured, and which could emit GW signals large enough to be eventually detectable were binary systems made of two compact objects as first envisaged by Freeman Dyson. This motivated a worldwide theoretical effort to predict, with as much accuracy as possible, the form of the GW signal emitted by compact binaries. Indeed, the largest amplitude of the GW signals expected from such sources was a few orders of magnitude smaller than the broadband noise of interferometric detectors (at their design sensitivity). In order to detect – and most importantly, to interpret and analyze – GW signals from compact binaries, a method of optimal filtering in which one crucially uses the a priori knowledge of the GW signal to dig it out of the broadband noise must be used. (This is similar to the commonsensical idea that in order to find a needle in a haystack, it is very helpful to know the size, shape, and constitution of the needle!)

The challenge here was both to be able to keep track, with high accuracy, of the slow evolution of the frequency, phase, and amplitude of the emitted GW signal during the several thousands of orbital periods where the two stars get «closer with ever-increasing speed» (inspiral part of the GW signal), and to be able to predict the final part of the GW signal, that is, as Dyson said, the «gravitational flash ... of unimaginable intensity» emitted during the merger of the two compact objects. In particular, it was realized that, if black holes with thirty or forty times the mass of the Sun existed in binary systems, they would be the strongest GW sources, because they would coalesce at a few hundred cycles per second (i.e., close to the LIGO-Virgo detector's sweet spot). Several groups around the world worked hard on both aspects of this challenge. An increased knowledge of the inspiral part of the GW signal was obtained through refined computations based on analytically solving Einstein's field equations [22–25]. However, the authors of reference [27] emphasized that the GW phase would need to be correct to within half a cycle or so during the entire frequency sweep, and that the usual PN *analytical* descriptions of the inspiral GW signal available at the time were losing their accuracy several orbits before the merger of the two objects. The mathematical properties of the PN series of the two main ingredients (i.e., the binary-system's energy and GW flux) entering the GW phase were also unclear (were the PN series convergent or asymptotic?). It was thus concluded [28] first that there was no hope of using analytical methods for describing the full inspiral signal up to the merger, and *a fortiori* no hope of getting an analytical description of the complete GW signal emitted by coalescing binary black holes.[4] Another conclusion was that it was necessary to develop supercomputer-based numerical simulations of the merger of binary black holes to have a sufficiently accurate description of the complete

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[4] The problem of coalescing binary black holes involves solving Einstein's equations without any material source and is therefore a "cleaner" problem than coalescences involving neutron stars, where one has to tackle the relativistic hydrodynamics of nuclear matter.

GW signal. However, solving Einstein's equations by numerical methods in the case of a binary system of two black holes turned out to be more difficult – and delicate – than initially expected. In spite of many years of work and the availability of supercomputers, no numerical way of predicting the GW signal emitted by binary black holes existed at the end of the 1990s.

Damour saw this situation as an interesting intellectual challenge that could motivate the development of new analytical approaches. The main driving idea was to look for ways to extend the domain of validity of an analytical description of the inspiral GWs by combining several *resummation* techniques.[5] In the summer of 1997, collaboration with Bala Iyer and B. S. Sathyaprakash [29] introduced two such resummation techniques: (i) one (which was inspired by Damour's reading of Itzykson and Zuber's book on quantum field theory [30]) suggested transforming the usual function describing the total energy of the two-body system into another energy function which had proven useful in quantum electrodynamics «when trying to extend one-body-in-external-field results to two-body results"»; (ii) the second used Padé approximants to extend the domain of applicability of the function describing the instantaneous flux of gravitational radiation.

The first prediction of the complete gravitational-wave (GW) signal was emitted when two black holes spiral around each other before finally merging. In September 1997, Alessandra Buonanno arrived for

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[5] "Resummation" is a general word which refers to mathematical transformations allowing one to extend the domain of validity of naive perturbative expansions (of the type called Taylor expansions).

a two-year postdoctoral position at the Institut des Hautes Etudes Scientifiques (IHES) and began a fruitful collaboration with Damour. This collaboration led, in particular, to the invention of a new analytical approach to the general relativistic two-body problem called the Effective One-Body (EOB) approach. The development of the EOB approach occurred in two stages: first, during the summer-autumn of 1998 [31] and the spring-summer of 1999 at IHES, and then at a distance[6] during the autumn-winter of 1999-2000 [32].

In November 1997, the second Gravitational-Wave Data Analysis Workshop was held in Orsay, near IHES. Buonanno, who until then had worked in the theoretical cosmology of the early universe, was lucky to attend the workshop and be exposed to cutting-edge GW research, including the multi-talk session dedicated to the first attempt to search for inspiraling binaries with the 40-meter interferometer's data at Caltech. Her involvement in GW research was also boosted by several personal encounters she had during the workshop, including among others Luc Blanchet, Eric Poisson, B. Sathyaprakash, Bernard Schutz, and Kip Thorne. Ultimately, she was struck and fascinated by the amount of theoretical and data analysis work that was still required to make GW astronomy a reality.

Without entering into technical details, here are the main ideas and initial results of the EOB approach. For simplicity, focus is placed on the case of the coalescence of two non-rotating black holes, for which

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[6] Alessandra Buonanno left IHES in October 1999 to start the Richard C. Tolman Fellowship in Kip Thorne's group at Caltech.

the EOB method yielded especially useful results.[7] The EOB formalism is guided by the notion that non-linear (or non-perturbative) effects can be captured analytically if the key ingredients that enter the two-body dynamics and GW emission are properly resummed, using hints from the (exact) test-body/limit results. The four main ingredients of the EOB description [31, 32] of the GWs emitted by binary black holes can be summarized in the following four points. (1) The conservative[8] dynamics of a two-body system, with masses $m1$, $m2$, is mapped onto the dynamics of a fictitious body of mass $\mu \equiv m1m2/(m1 + m2)$ moving in some «effective» spacetime equipped with a generalized Riemannian structure (of the Finslerian type), which is resummed in an appropriate way. (2) The radiation-damping effects are described by adding a resummed radiation-reaction force to the conservative Hamiltonian equations of motion. (3) The emitted GWs are also described by a suitably resummed version of the GWs that would be emitted by a binary system whose relative motion, in the center of mass, can be identified with the motion of the fictitious body of mass μ. (4) The effective one-body dynamics is used to describe the motion and radiation of the two-body system up to the (EOB- defined) *merger*, and is then completed into a full waveform by smoothly matching the pre-merger waveform to a post-merger, *ringdown* signal, made of a superposition of quasinormal modes [12] of the final black hole formed

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[7] Note, however, that the EOB approach also led to highly accurate descriptions of the GW emission from binary neutron stars, or neutron-star black-hole systems, up to the merger (see, e.g., Refs. [34] [36]). The post-merger signal of binary neutron stars is more complicated to describe and requires detailed numerical simulations.

[8] “Conservative” refers to a time-symmetric dynamics where all time-antisymmetric effects linked to radiation damping are neglected.

by the merger (the mass and spin of the final black hole being determined, at least in the early work, in terms of the initial masses and spins, by using the energy and angular-momentum of the system close to the EOB-defined merger). Note that the initial versions of the EOB approach, say until 2005, were completely autonomous, in the sense that all the ingredients (including the mass and spin of the final black hole formed after merger) were analytically determined within the EOB approach itself. As discussed below, later versions of the EOB approach used results extracted from numerical relativity simulations to improve the accuracy of the EOB-defined waveform.

In early 2000, the EOB approach gave the first estimate of the complete gravitational waveform (covering both inspiral, merger, and ringdown) emitted by the coalescence of two non-spinning black holes [32], as illustrated in Fig. 1 (see Ref. [33] for the extension to the coalescence of spinning black holes). In the figure, the gravitational waveform (h) is represented as a function of time (t). As it passes through an interferometer with two arms of length L ($L = 4(3)$ km for the LIGO (Virgo) interferometers), the physical effect of this waveform is to cause a time-dependent change δL(t) in the differential mode of motion of the arms on the order of $\delta L(t) = \pm h(t)L/2$ (for the optimal orientation of the interferometer with respect to the incident wave). The left part of the graph of $h(t)$ (in red) represents the GW emitted during the inspiral and the «plunge» (i.e., the last couple of orbits before the merger), whose beginning is approximately indicated by several colored symbols in the figure. The merger corresponds to the end of the red curve (occurring at the rescaled time $t/M \sim 100$). The right part of the graph (hyphenated and blue) represents the ringdown signal, made of a superposition of quasinormal modes (only). Note that the typical GW amplitude currently observed by the three LIGO-Virgo interferometers reaches, at its maximum, a value of order $h_{\max} \sim 10-21$. Such a small value means that the length of the arms of the interferometers vary at most by $\pm 2 \times 10-16$ cm, which is about 100 million times smaller than the size of an atom. This smallness makes one appreciate the extreme challenges that experimenters had to overcome to succeed in detecting GWs. As theorists, Buonanno and Damour are happy that their work has provided an additional stone to the scientific monument that the successful detection and data analysis of GWs represent.

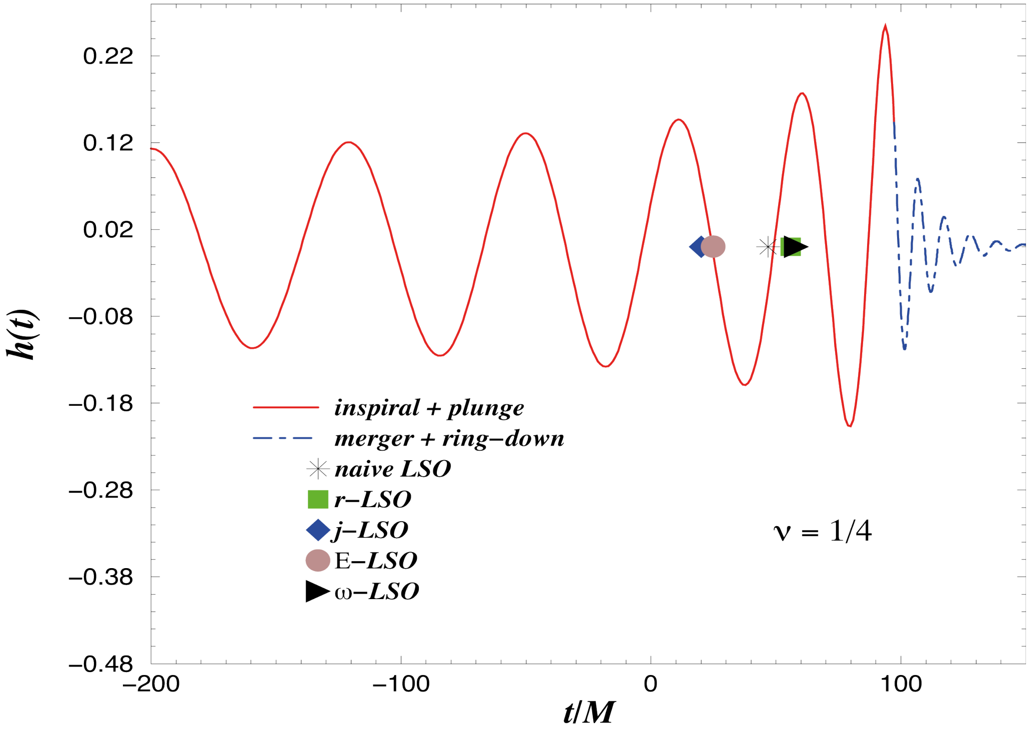

Fig. 1. First estimate of the complete gravitational waveform from an equal-mass binary black hole, moving along a quasicircular orbit, using the effective-one-body formalism. (The figure is adapted from Ref. [31].)

When the EOB method was set up, the predictions it made (such as Fig. 1) for the complete waveforms emitted by coalescing binary black holes could not be compared to any other estimates, because no other analytical method that could tackle this problem existed, while numerical approaches had failed to set up stable codes that could simulate inspiraling and merging black holes (see, however, the pragmatic, hybrid *Lazarus* approach [37], which aimed at predicting the plunge and merger waveform). As in the discussion to follow, a decisive moment took place in 2005, thanks to breakthroughs in numerical relativity, starting with the pioneering work of Frans Pretorius [38].

The original analytical EOB waveform had to be changed to the improved EOBNR waveforms used for the data analysis and parameter estimation of LIGO-Virgo data. In the late 1990s, while the constructions of the two LIGOs in the United States, Virgo in Italy, GEO-600 in Germany and TAMA-300 in Japan were underway, the plan to upgrade LIGO and Virgo to advanced configurations had already been undertaken and was considered crucial for the first observation of GWs from compact binary systems. Meanwhile, following the vision of Barry Barish, the LIGO Scientific Collaboration (LSC) was established, with Rai Weiss as the first spokesperson. The goal was to bring together theorists, data analysts, and experimenters from the main GW groups in the United States, the United Kingdom, and Germany. As a young postdoc at Caltech in the fall of 1999, Alessandra Buonanno was lucky to find herself immediately at the core of several, vibrant research activities, encompassing (local) GW astrophysicists, experimenters, and data analysts[9]. She learned first-hand, engaging in discussions with them, and working with graduate students in Thorne's group (Yanbei Chen and Michele Vallisneri), mainly on the intricacy of applying theoretical predictions to GW observations. She also gained an understanding of the sophistication of the experiment and of the detection process, working on quantum optical techniques to overcome the free-mass standard quantum limit in advanced LIGO and Virgo detectors [39, 40], and on templates to account for possible inaccuracies of the PN predictions for the late inspiral, and for the drop in signal-to-noise ratio (SNR) if the merger signal was absent [41, 42]. Work to characterize and simplify the modeling of spin precession effects (comparable to a gyroscope or a spinning top) in the GW signal from binary black holes was considered decisive for the correct interpretation of GW signals [43], and was also undertaken in those years [42, 44]. An important step forward was the introduction of the *precessing reference frame*, where the waveforms exhibit relatively smooth amplitude and phase evolutions and are well approximated by nonprecessing waveforms. The waveforms in the inertial (observer) frame are then obtained from the ones in the *precessing frame* by a mere rotation (or twist). Since the time when those experiments were carried out, this method, which has drastically simplified the modeling of precessional effects, has been improved [45–47] and adopted by all current waveform models that include spin-

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[9] Buonanno joined the LSC in 2000 through Thorne's Memorandum-of-Understanding (MOU) while she was at Caltech and then at the CNRS in Paris. In 2005, she established a MOU between the LSC and her group at the University of Maryland, and subsequently at the Max Planck Institute for Gravitational Physics in Potsdam.

precession effects, as well as the LIGO and Virgo detectors. In those years, a numerical relativity group was established between Caltech (Kip Thorne) and Cornell (Saul Teukolsky), eventually giving rise to the Simulating eXtreme Spacetime (SXS) collaboration. Buonanno directly learned about the progress that the numerical relativity group was making on the binary/black hole problem by attending Thorne's weekly group meetings. She was able to establish important connections with the SXS project, which will play a central role in her scientific career in the years to come.[10]

Meanwhile, after the initial analytical derivation of the full waveform emitted by a binary/black hole coalescence [32] in early 2000, several works extended the scope of the EOB formalism in 2000-2001. The EOB conservative dynamics was augmented with 3-PN terms in Ref. [48], while the first extension to the physically important case of spinning black holes was done in Ref. [49]. However, in the absence of other theoretical predictions against which to compare EOB waveforms, the LSC Compact-Binary-Coalescence group made the (conservative) decision of not employing EOB waveforms with merger-ringdown signals for the first search of non-spinning binary black holes with LIGO data. In fact, the very first search of this kind [50] was restricted to the inspiral, and used waveforms with phenomenological coefficients [41], which spanned differences between PN and EOB

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[10] Quite interestingly, Frans Pretorius started a Richard C. Tolman Fellowship at Caltech in Thorne's group in the fall of 2002, a few months after Alessandra Buonanno left Caltech to join the CNRS in France. Eventually, the two met for the first time in the fall of 2005, at a NASA-Goddard workshop in Maryland.

theory, and possible inaccuracies in PN theory in the late inspiral and in the transition from inspiral to plunge.

In the fall of 2001, Buonanno was hired by the Centre National de la Recherche Scientifique (CNRS), first at the Institut d'Astrophysique de Paris and then at the Laboratoire Astroparticule et Cosmologie in Paris. While at Caltech, she appreciated the importance of modeling spin effects in binary systems and established a fruitful collaboration with the analytical relativist Luc Blanchet. In those years, the knowledge of the two-body dynamics in the presence of spins was pushed through 2.5-PN order [51–54]. The first (purely analytical) EOB-based estimate of the complete GWs emitted by systems of two spinning, precessing black holes was given in Ref. [33], demonstrating the robustness of the EOB method in the quasi-circular, generic-spin case.

A turning point in GW modeling occurred in 2005, when after more than thirty years of attempts, the first numerical relativity simulation of a binary black hole at last unveiled the merger signal. This was reported in a single author paper by Frans Pretorius in the summer of 2005 [38]. Buonanno attended a workshop at the Aspen Center for Physics in May-June 2005, LISA Data: Analysis, Sources and Science (organized by Vicky Kalogera and Alberto Vecchio). There, Pretorius's results, which had already been presented at conferences in the previous months, were closely scrutinized and vividly discussed. Subsequently, in November 2005, the workshop Numerical Relativity 2005: Compact Binaries was organized at the NASA Goddard Space Flight Center (GSFC), near the University of Maryland (UMD). In late August 2005, Buonanno took over a faculty position at UMD and was immediately invited to the workshop to present the EOB work for binary spinning black holes [33].

At the same time, two other independent, numerical relativity groups (from NASA's GSFC and the University of Texas in Brownsville) presented the results of their binary/black hole simulations for the first time [55, 56], thus confirming Pretorius's findings. Eager to understand the first inspiral-merger-ringdown numerical simulations and to compare them to the EOB-based waveforms, Buonanno had several personal encounters with numerical relativists at the workshop and started a very valuable collaboration with Greg Cook and Frans Pretorius. This collaboration led to the first comparison of an EOB-based, complete GW with one from a numerical relativity simulation, but it also offered a more comprehensive understanding of the role of the quasinormal modes in the transition from merger to ringdown [57]. Several predictions of the EOB approach were broadly confirmed by the numerical results, including the blurred, adiabatic transition from the inspiral to the plunge, the extremely short merger phase, the simplicity of the merger waveform (i.e., the absence of high-frequency features in it), and the estimates of the radiated energy during the last stages of inspiral, merger, and ringdown (including the EOB-based predictions of the final mass and spin). Over the next few years, synergies among numerical and analytical relativists blossomed. Effective-one-body waveforms were further improved and completed through numerical relativity information. To achieve the high accuracy required by LIGO and Virgo analyses, the EOB waveforms had to be calibrated to a discrete number of numerical relativity simulations and then extrapolated to the entire binary parameter space [58, 59]. In parallel, fast to generate, frequency-domain, closed-form phenomenological waveform models were also developed [60–62] by combining EOB waveforms at low frequency and numerical- relativity ones at high frequency. The first search of non-rotating binary/black-hole coalescences with LIGO and Virgo data, collected between 2005-2007 [63] and 2009-2010 [64], employed the first generation of EOBNR[11] templates [58].[12]

The next important step, in view of the first, more sensitive advanced LIGO observing run, which was expected to take place in the fall of 2015, was to extend the EOBNR template bank to spinning black holes (thus, SEOBNR). Furthermore, to make this template bank highly accurate (as was crucial for the identification of the astrophysical sources, via inference studies upon detection) careful calibration of these spinning black hole templates to numerical relativity simulations was called for. With her group at UMD (notably Enrico Barausse, Yi Pan, and Andrea Taracchini), Buonanno developed an EOB theory for spinning black holes that employed a fictitious test-spin [65–67] instead of a test-mass [49, 54], and used it to build faithful SEOBNR waveform models calibrated to the highly accurate numerical simulations of the SXS collaboration [46, 68, 69].

On the other side of the Atlantic, a parallel effort to improve the EOB theory and waveforms was pursued by Thibaut Damour and Alessandro Nagar in collaboration with several European numerical relativity experts (notably Luciano Rezzolla, Bernd Brügmann, and Sebastiano Bernuzzi) [70–73]. In collaboration with A. Nagar and Bala Iyer,

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[11] The name EOBNR was chosen to emphasize that the EOB waveforms are completed with numerical relativity (NR) information.

[12] The use of EOBNR waveforms for the first search of binary black-hole mergers highly benefited from a fruitful collaboration between AB's group and Sathyaprakash's group at the University of Cardiff.

Damour developed a novel resummation of the radiation-reaction force and waveforms [74], which was extended to spin effects in Ref. [75], and employed in the SEOBNR waveform models [46, 69]. Deeper understanding of the transition from the late-inspiral to merger and ringdown in the test-body limit [76, 77] was also highly beneficial to guide the construction of accurate waveforms for comparable-mass binary systems (i.e., of the kind LIGO and Virgo observe). In parallel, phenomenological templates were also improved and extended to capture spin effects [45, 47, 62, 78, 79]. Moreover, in 2011, Buonanno established and co-led (with Duncan Brown and Frans Pretorius) the Numerical-Relativity – Analytical-Relativity (NRAR) collaboration [80], an international project supported by the National Science Foundation that was aimed at producing numerical relativity simulations of binary black holes and at using them to compare and improve analytical waveform models to be used in advanced LIGO and Virgo observing runs.

In the fall of 2014, Buonanno moved back to Europe, to the Max Planck Institute for Gravitational Physics in Potsdam, where she established ex novo an interdisciplinary research group which spanned from theory (numerical and analytical) to observation through the analysis of experimental data. From day one, its aim was to be at the forefront of GW astronomy, when the advanced LIGO detectors would come online in the fall of 2015. During 2014-2015, LIGO and Virgo data analysts (including Ian Harry in Buonanno's group) developed the so-called über template bank for the upcoming observing run. It used about 75,000 PN templates for binary neutron stars, and about 350,000 SEOBNR templates to cover the neutron star–black hole and binary blackhole parameter space with total masses between four and one hundred times the mass of the Sun, and mass ratios up to ninety-nine (see, e.g., Ref. [81]). After that, one of the greatest scientific discoveries of this century took place.

On 14 September 2015 at 9:50:45 Coordinated Universal Time (UTC) a GW signal launched by two black holes merging at about one billion light-years away passed by the advanced LIGO detector in the state of Lousiana, causing a variation in the interferometer's differential mode of motion of about one-hundred-millionth of an atom's size. The event was then recorded, about 7 msec afterwards, at the twin LIGO detector in the state of Washington. The event was dubbed GW150914 [82].[13] This historic event successfully crowned fifty years of experimental and theoretical work in GW physics, ushering in the era of GW astronomy. GW150914 was an unexpectedly loud event, detected with a signal-to-noise-ratio (SNR) of ~ 24. Its loudness allowed it to be initially identified by an (online) generic transient search, which used minimal assumptions about waveforms. The highest statistical confidence was obtained with the (offline) optimal filtering searches that employed 350,000 SEOBNR waveforms (see above). Inference studies with waveform models identified the signal as a binary composed of two black holes with 36 and 29 times the mass of the Sun. The energy released by the binary, as it spiraled in and merged, was stupendous: in total three solar masses were radiated away in GW energy. The optimal filtering searches were essential for detecting the second, less loud but

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[13] Gravitational-wave signals are named using the day, month and year in which they are detected.

longer, signal observed by LIGO [83] during the first run: GW151226. By contrast to GW150914, whose SNR ~ 24 is spread over 0.2 sec (~ 10 GW cycles), the SNR ~ 13 of GW151226 is spread over about 1 sec (~ 55 GW cycles).

Buonanno was filled with wonder and disbelief when she saw the first reconstruction of the signal with the SEOBNR waveform models appear on the computer screen. Meanwhile, there was also some tension among herself, her group, and the entire LSC and Virgo Collaboration as they had to figure out whether what they had observed was a GW from the deep, dark universe, or simply random fluctuation of the noise in the two LIGO detectors. Moreover, several studies were carried out to exclude that GW150914 was a fluke, and the observation of the second event, GW151226, a few months later was certainly a reassurance. A few numerical groups within the LSC run numerical relativity simulations with parameters close to the ones reported by the inference study were done with waveform models. In Buonanno's group, which was part of the SXS collaboration, it took three weeks for Serguei Ossokine to perform the run and compute the gravitational waveform for the event GW150914, and three months for the much longer event GW151226. The very good agreement of the SEOBNR waveforms with the numerical simulations confirmed that the GW signals had been properly identified by the waveform models at their disposal.[14] Buonanno was honored to attend the press conference on

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[14] In early 2016, at the end of the teleconference where the LSC and the Virgo Collaboration decided to finally submit the GW150914's paper to the journal, Physical Review Letters, AB and her group at last celebrated drinking champagne! That was accompanied by a bit of nostalgia in losing the exclusivity of the discovery, which had kept everybody extremely busy in the previous four months.

February 11, 2016 (followed online by Damour) in person in Washington DC, where the discovery of GW150914 was eventually announced, and at the end of which the LSC spokesperson, Gabriela Gonzalez, briefly acknowledged the importance of «analytical waveforms».

Since 2015, the LIGO and Virgo detectors have observed about 100 GW signals from *compact binary systems* [84], including the first binary–neutron star coalescence in 2017 [85], GW170817, which was followed by the observation of electromagnetic signals in the gamma, X-ray, optical, infrared, and radio wavelengths. The combined observation with the Fermi Gamma-Ray Space Telescope made it possible to solve the thirty-year-old puzzle about the origin of short bursts of high energy radiation in the cosmos, ushering in the field of multi-messenger astronomy with gravitational waves. In early 2020, GW200115, the first robust mixed binary (i.e., a binary composed of a black hole and a neutron star) was discovered [86]. Among others, the LIGO and Virgo observations have provided us with the most convincing evidence to date that dark compact objects from five up to ninety times the mass of the Sun are described by black holes, i.e., by the solution of the Einstein's equations found by Karl Schwarzschild in 1916 in the non-rotating case [2], and by Roy Kerr in 1963 in the rotating case [87]. No deviation from the theory of general relativity has been found in GW observations thus far [88].

In the next decade, the European Space Agency's (ESA) large-class mission LISA [89] (also with important contributions from NASA) will fly (~ 2035) and open a new bandwidth of the GW spectrum between

0.0001 Hz and 0.01 Hz. It will observe astrophysical sources that complement the ones seen on the Earth, notably supermassive binary black holes, extreme mass ratio inspirals composed of a stellar-mass compact object spiraling into a supermassive black hole, and the astrophysical stochastic GW background produced by double white dwarf binaries in our galaxy. Quite interestingly, binary black holes of the kind currently detected by LIGO and Virgo, or heavier, will also be observed by LISA for days or months, a few years in advance of ground-based facilities (depending on the binary parameters). On the ground, new facilities such as the Einstein Telescope [90] in Europe and the Cosmic Explorer [91] in the United States have been planned for the next decade. They aim at improving the noise spectral density of advanced LIGO and Virgo by one or even two orders of magnitude depending on the frequency, broadening the bandwidth down to3–5 Hz. Among the scientific goals, they will be able to observe binary black holes from the time the first stars formed, when the Universe was about ten times smaller. All these future observatories will give us the possibility to carry out the most exquisite tests of general relativity in the highly dynamical, strong field regime, thereby challenging our knowledge of gravity and nuclear physics, and unveiling the astrophysical origin and environment of black holes and neutron stars in our universe. To take full advantage of this unprecedented discovery potential, continuing to make predictions of ever more accurate gravitational waves will be instrumental!

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